The Psychodynamics of the Numbers^{*}

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I. Introduction

In his book *Platonism and Anti-Platonism in Mathematics*, Mark Balaguer argues that the only viable philosophies of mathematics are *fictionalism* and *full-blooded Platonism*, but that no fact of the matter can decide between them.¹ Therefore we are free to accept full-blooded Platonism, which is the view, roughly speaking, that any consistent mathematical system is true, and therefore that the mathematical objects in such systems exist in a Platonic sense.² I will argue here that some of these mathematical objects have in addition an objective psychological reality, for they are rooted in the neurodynamics of the brain and structure human psychodynamics. These are the mathematical structures that are potent and numinous in human experience. They include, as I will explain, the archetypal numbers, such as the Monad, Dyad, Triad, and Tetrad, and fundamental geometrical forms, such as circles, triangles, and mandalalike figures, which are the basis of the use of *kharaktêres* as *sunthêmata* in theurgy. The archetypal numbers also include fundamental intuitions of discreteness and definiteness, as opposed to continuity and indefiniteness. I will argue further that certain numbers, as innate, unconscious psychodynamical Forms inherent in the human psyche, have qualitative psychological effects in addition to the quantitative properties studied in mathematics. Indeed, this is the valid core of Pythagorean arithmology, as found for example in the anonymous *Theologumena arithmeticae (Theology of Arithmetic).*³ Therefore, a complete contemporary Platonic philosophy of mathematics must comprehend the inherent qualitative as well as the quantitative aspects of the archetypal numbers.

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¹ Balaguer (1998) 4–5; see also ch. 8.

² Balaguer (1998) 5.

 $^{^{3}}$ The Greek text is cited by page and line number from the de Falco edition, henceforth *TA*. Translations are from Waterfield (1988) unless otherwise stated.

The *Theologoumena arithmeticae*, on which this paper focuses, is dated to the mid-fourth century CE^4 and sometimes was attributed to Iamblichus, which is incorrect. It is a compilation of extracts from Nicomachus' *Theologumena arithmeticae*, otherwise surviving only as a summary in Photius' *Biblioteca* 187, and extracts from Anatolius' *De decade*, as well as other material.⁵

To understand the universal qualitative aspects of certain numbers, it is necessary to make a short digression through the neuropsychology of number. Human perception of small numbers is just as innate as is our perception of shape, color, size, and spatial relation. In common with some other animals, we have an innate number sense; that is, we can directly perceive the number of objects in a group without explicit counting.⁶ For humans this is limited to six or seven objects unless the objects are arranged in a regular pattern. We can also perceive numerosity in sound or light sequences up to three or four.⁷ In general, our number perception is correct up to four and its accuracy decreases linearly up to about eight; reaction time is about 600 milliseconds up to four objects and increases linearly to 800 milliseconds for about seven objects.⁸ These are not learned skills, for infants from four days to a few months old already exhibit number perception;⁹ they also exhibit *innate arithmetic*, that is, the ability to recognize sums and differences, for numbers up to three or four.¹⁰

Recent neuroimaging studies have identified a region in the brain, in the right posterior parietal cortex, in which small numbers are represented.¹¹ It is approximately rectangular, 2×3 cm in size, with the areas that respond to the numbers one to seven arranged in order; more area is devoted to the smaller numbers, which are represented with greater accuracy. Similar regions have been found in non-human primates and crows, which also exhibit number perception.¹² It has been established that these neurons are responding to the *numbers* of objects in a group, and not to other characteristics, such as their individual size, spatial extent, density, arrangement, shape, size, or total area.¹³

⁴ Waterfield (1988) 23.

⁵ Waterfield (1988) 23; O'Meara (1989) 15n24.

⁶ Lakoff and Núñez (2000) ch. 1, 51. Butterworth (1999a) and (1999b) are popular introductions to the topic of innate arithmetic and numerosity. They appear to be identical, but with different pagination. For a dissenting view, see Núñez (2017) and a reply Nieder (2107).

⁷ Lakoff and Núñez (2000) 16, 21–22.

⁸ Lakoff and Núñez (2000) 19–20.

⁹ Lakoff and Núñez (2000) 16.

¹⁰ Lakoff and Núñez (2000) 15–18.

¹¹ Harvey, Klein, Petridou, and Dumoulin (2013). They identify the specific region as in the "posterior superior parietal lobule, centered at mean (SD) Montreal Neurological Institute x, y, z coordinates of 23 (4), -60 (7), 60 (7) (18)," in Brodmann area 7. There is a similar area in the left hemisphere, but it does not have an equally precise representation of numerosity.

¹² Wagener, Loconsole, Ditz, and Nieder (2018).

¹³ Harvey, Klein, Petridou, and Dumoulin (2013).

Therefore we have an innate direct perception of *numerosity*, the number of objects in a group, but we can also perceive *equinumerosity*, the one-to-one correspondence between the objects in two groups, up to about sixteen.¹⁴ In addition, by the age of fifteen months, infants understand numerical order.¹⁵

Our innate qualitative understanding of number also involves our direct perception of symmetry, which is exhibited by infants and some non-human animals.¹⁶ The most salient symmetry is *reflection* or *mirror symmetry*, which can be recognized in 100 milliseconds.¹⁷ A possible evolutionary explanation of its salience is that it often betrays the presence of an animal, for animals have external bilateral symmetry to facilitate motion and perception on the surface of the earth.¹⁸ Moreover, perceiving a mirror symmetry, such as paired eyes, can indicate that you are the focus of attention of another animal, which is important for survival.¹⁹ Vertical axes of symmetry are the easiest to perceive, but reflections across other axes are also easily perceived in context.²⁰ In addition we are able to perceive directly other kinds of symmetry, such as *translation symmetry*, in which an object is repeated one or more times, and *rotational symmetry*.²¹

So far, we have considered our innate ability to *perceive* certain small numbers and symmetries, but we also have the ability to *imagine* them. For example, we can easily imagine regular arrangements of small numbers of objects. We find these in the Pythagorean *figurate numbers*, in which some number of dots are arranged in a triangle, square, rectangle, pentagon, or other shape; they exhibit mirror, rotation, and translation symmetry. I believe we can easily visualize figurate numbers up to about nine, and higher numbers in a few special cases such as twelve, sixteen, and twenty. Other common ways of imagining the numbers are as regular convex polygons, such as pentagons and hexagons, and regular star polygons, such as pentagrams and hexagrams. I think these are easily visualized up to about eight. These natural and probably innate ways of visualizing numbers are their archetypal images.

Psychologically, besides the passive aspects that we have considered so far, the archetypal numbers have active aspects. In their passive aspect, numbers appear as archetypal images in perception and imagination, but like other archetypes, the numbers are fundamentally subconscious dynamical forms that regulate perception, affect, motivation, and behavior. That is, the archetypal numbers are psychodynamically active, which will be a central topic in this article.

¹⁴ Crossley (2007) 82.

¹⁵ Lakoff and Núñez (2000) 18.

¹⁶ Tyler (2002) 25.

¹⁷ Tyler (2002) 14, 26.

¹⁸ Tyler (2002) 4–6.

¹⁹ Tyler (2002) 8.

²⁰ Tyler (2002) 31, 44.

²¹ Tyler (2002) 12, 26–28.

How far can we extend this account of the archetypal numbers? Some people argue that the archetypal numbers do not extend beyond the Tetrad.²² Based, however, on neuropsychological evidence and the numerological tradition, I'm inclined to think they extend a bit larger, but are likely encompassed in the Decad, or perhaps the Dodecad.

C. G. Jung considered the archetypal numbers to be fundamental to depth psychology, remarking that number "may well be the most primitive element of order in the human mind."²³ He assigned their investigation to Marie Louise von Franz, and in her book *Number and Time* she explains that numbers are not mere quantities, but have four principal aspects (which form a quaternity):²⁴

I. Quantitative II. Geometric III. Algebraic IV. Qualitative

Qualitatively, each of the archetypal numbers has an individual character.²⁵ She quotes Henri Poincaré, who said, "Every whole number is detached from the others, it possesses its own individuality, so to speak."²⁶ The first four numbers in particular stand out for their rich archetypal structure. Karl Menninger claims that in all languages the words for one to four are etymologically adjectives, implying that they are qualities, but the words for larger numbers are not adjectives.²⁷ Likewise, von Franz claims, Plato recognized the archetypal character of these numbers by using substantives in $-\alpha \zeta$ such in *monās*, *duās*, *triās*.²⁸ Theon of Smyrna says these are the intelligible principles for the sensible numbers, which are denoted by adjectives such as *hen*, *duo*, *tria*, and *tettares*.²⁹ The difference is between essence and quantity.

There is no explicit ontology of numbers in the *Theologumena*, but some relevant aspects of it can be inferred from Nicomachus and Iamblichus.³⁰ Ordinary "scientific" (*epistêmonikos*) mathematics was discussed in the "minor arithmetics," such as Nicomachus' surviving *Introduction to Arithmetic*, Iamblichus' *On Pythagoreanism* IV, and Anatolius' *Arithmetical Introductions* (surviving only in fragments). These dealt with mathematics as intermediate between the sensibles and the Forms and reflect mathematics as a ladder to draw the inner eye up to pure being. However, Nicomachus seems to have taken the Numbers themselves as prior to the Forms and indeed as their principles, the highest form of being. The Forms then are

²² Crossley (2007) 82.

²³ Jung (*CW* 8) ¶870.

²⁴ von Franz (1974) 75–76.

²⁵ von Franz (1974) 60–61, 101n1.

²⁶ von Franz (1974) 60.

²⁷ Menninger (1992) 18–32. See also von Franz (1974) 114–115.

²⁸ von Franz (1974) 67n19; *Phaedo* 101C3–4, 105C10.

²⁹ Theon Smyrnaeus, *Exp. rerum math.* (Hiller) 20, 5–11. See also Kalvesmaki (2006) 286, 290.

³⁰ Here I follow O'Meara (1989) 16–22, 76–85.

properties or characteristics of the Numbers and subordinate to them. These divine Numbers (transcending the intelligible, ideal numbers) were treated in the "major arithmetics," such as his *Theologumena*, Anatolius' *De decade*, the Pythagorean *Sacred Discourse*, Iamblichus' *On Pythagoreanism* VII,³¹ and our anonymous text, where the numbers are assimilated to gods and apprehended through "higher insight" (*kata kreittous ennoias*).³² In consequence, the higher arithmetic serves the role of dialectic in Plato's *Republic*.

Von Franz says that, like other archetypes, the numbers manifest distinctly in consciousness, but that they interconnect and interpenetrate each other in the unconscious. She calls each a *hen-to-pan* to signify that each number is an aspect of the numbers as a whole; they are all differentiated aspects of the *One-continuum*.³³ The natural numbers are genuine symbols, pregnant with meaning, capable of activating the archetypes from which they emanate.³⁴ They have inherent dynamism for, like all archetypes, they are innate forms of activity.³⁵ Likewise, they are autonomous, for the archetypes can behave like independent beings,³⁶ and thus it is not unreasonable that the *Theologumena* identifies numbers with gods, or that gods descend from the *henads*. Therefore, von Franz says, "each number must be thought of as containing a specific activity that stands forth like a field of force."³⁷

Each of the archetypal numbers has an individual character, but the first four numbers are preeminent for their rich psychological structure. Psychologically, according to von Franz, "One comprises wholeness, two divides, repeats, and engenders symmetries, three centers the symmetries and initiates linear succession, four acts as a stabilizer by turning back to the one as well as bringing forth observables by creating boundaries."³⁸ Here I will address them in order, comparing their psychodynamical structure as archetypal numbers with their descriptions in the *Theologumena*. Certainly, some of its associations are based on false etymologies and simple numerical relationships, but we will see there is a core of insight into the archetypal numbers.

II. Monad

We begin at the beginning. The Monad is paradoxical in its unity, as attested by both the *Theologumena* and depth psychology. For example, the *Theologumena* says that the Monad contains everything potentially, therefore it comprises many things that are opposed or contrary

³¹ O'Meara (1989) 76–85.

³² O'Meara (1989) 228–229, quoting Iamblichus in Psellus.

³³ von Franz (1974) 62, 65–66.

³⁴ von Franz (1974) 73–74.

³⁵ von Franz (1974) 73–74.

³⁶ von Franz (1974) 74.

³⁷ von Franz (1974) 75.

³⁸ von Franz (1974) 74.

in actuality.³⁹ This paradoxical character is consistent with the Neoplatonic Ineffable One, but also with the Jungian Self, which is "the central core of the personality"; that is, "our inborn individuality *and* the process by which that individuality seeks to be realized in our life."⁴⁰ Jung says that the Self "is absolutely paradoxical in that it represents in every sense thesis and antithesis, and at the same time synthesis"; it is "a union of opposites par excellence."41 Moreover the Self is an unconscious unity, a wholeness without reflection.⁴²

Since, according to the *Theologumena*, the Monad contains everything potentially, including all the things that are opposed in actuality,⁴³ it even produces the Dyad.⁴⁴ In particular, the *Theologumena* says the Monad is called "Androgyne" (*arsenothêlun*), for it is considered neither odd, that is, male, nor even, that is, female.⁴⁵ As the seed of all the numbers, it is neither male not female, because the seed precedes differentiation of the sexes.⁴⁶ The Monad is thus both father and mother and as such it is also the principle of both the form and matter in everything.⁴⁷ Depth psychologists likewise say the Self is hermaphroditic and compare it to the spherical original humans in Aristophanes' myth in the Symposium.⁴⁸

The *Theologumena* calls the Monad dark and obscure,⁴⁹ which is appropriate to both the Ineffable One and the paradoxical unconscious Self. It says the Monad is like the creative Chaos in Hesiod, in which all the unarticulated contraries are mixed in dark obscurity.⁵⁰ Because the Monad contains everything, it can be called the receptacle of all (*pandokheus*), and therefore the matrix $(gon\hat{e})$.⁵¹ Thus it is also called matter (*hulê*) as the source of the Dyad, which is more properly called matter.⁵² Psychologically, the Monad represents a state of uncritical unconsciousness in which a person submits naively to circumstances.⁵³ Nevertheless, this state is fertile; it wants to differentiate and develop.⁵⁴

According to the *Theologumena* the Monad generates both itself and everything else out of itself,⁵⁵ for it is self-producing and self-sufficient.⁵⁶ It adds that the Monad contains

³⁹ TA 3, 1–11.

⁴⁰ Sparks (2010) 54n46.

⁴¹ Jung (*CW* 12) ¶22. Citations to Jung's *Collected Works* (*CW*) are by paragraph number.

⁴² Sparks (2010) 60, 94, 105.

⁴³ TA 3, 2–5.

⁴⁴ TA 5, 14.

⁴⁵ *TA* 3, 21 – 4, 1; 4, 17 – 5, 2.

⁴⁶ *TA* 5, 5–10. See Waterfield (1988) for an emendation of de Falco's text here.

⁴⁷ *TA* 5, 2–5.

⁴⁸ Stevens (2003) 113, 226.

⁴⁹ *TA* 5, 18.

⁵⁰ *TA* 5, 16–19.

⁵¹ *TA* 5, 13, 20. ⁵² *TA* 5, 16–19.

⁵³ Sparks (2010) 105; von Franz (1974) 124.

⁵⁴ Sparks (2010) 112.

⁵⁵ TA 2, 15–17.

⁵⁶ TA 3, 17–18.

everything in potential form,⁵⁷ and therefore the Pythagoreans call it "Proteus," since he could assume any form.⁵⁸ Likewise all the archetypes are different manifestations of the Self.

Psychologically, as the beginning of the natural number series, the Monad is a metaphor for the beginning of anything.⁵⁹ The *Theologumena* compares it to the principle of the universe (*kosmikos logos*)⁶⁰ and to God as the source and regulator of everything.⁶¹ It is like Providence because it preserves all things.⁶² So also the unconscious Self, which Jung calls the God-image within,⁶³ regulates our psyches and guides our psychological development. J. Gary Sparks, a student of von Franz, says, "number is a prime representation of the spirit,"⁶⁴ which "is pulling us forward through the various stages of our life until we fill our potential."⁶⁵

The *Theologumena* continues that, as the form of forms (*eîdos eidôn*), the Monad is called Creation (*tekhnê*) on account of its creativity and is called Intellect (*noêsis*) on account of its intelligence (*noêtikos*).⁶⁶ As a creative principle, the Monad contains everything, but it does not deviate from its own principle, and so it is called "Atropos" after the Fate whose name means "the one who cannot be turned."⁶⁷ Similarly it is called Prometheus — commonly interpreted as Fore-thought — on the basis of an alternative etymology meaning that it does not outrun its principle.⁶⁸

The *Theologumena* calls the Monad *Nous* because it contains all the ideas that govern the cosmos⁶⁹ and therefore encompasses everything conceptually within itself.⁷⁰ It imparts sameness and constancy to the knowledge in the Nous.⁷¹ The *Theologumena* also identifies the Monad with moral wisdom (*phronêsei*), because what is correct is one.⁷² Anatolius remarks that the Pythagoreans call the Monad *Nous* and liken it to The One itself (*tô heni autên*), the intelligible god (*tô noêtô theô*), the uncreated (*tô agennêtô*), Beauty itself (*autokalô*), the Good itself (*autoagathô*), and among the virtues, the wisdom (*phronêsei*) of The One.⁷³ Psychologically, the ideas in the Nous are the transpersonal archetypes in the collective unconscious, which govern our psyches and are aspects of the Self.

⁵⁷ *TA* 1, 8–10; 3, 3–5. ⁵⁸ TA 7, 10–13. ⁵⁹ Lakoff and Núñez (2000) 75. ⁶⁰ TA 2, 17. ⁶¹ TA 2, 17–22. ⁶² TA 2, 20. ⁶³ Jung (*CW* 11) ¶¶282, 757. ⁶⁴ Sparks (2010) 50. ⁶⁵ Sparks (2010) 119. ⁶⁶ TA 2, 22–4. ⁶⁷ TA 4, 8–9. ⁶⁸ TA 4, 12–15. ⁶⁹ TA 3, 21–4.2. ⁷⁰ TA 4, 4–7. ⁷¹ TA 4, 3–4. ⁷² TA 6, 5–6. ⁷³ Anatolius, *De decade* (Heiberg) 29, 19–22. Cf. *TA* 6, 3–6. The Self is the integrative center of the psyche.⁷⁴ The *Theologumena* says that by virtue of its unity the Monad imparts coherence and harmony, and that it is in this respect like God; as a ruling principle, it is sun-like.⁷⁵ The Pythagoreans placed the Monad in the center of the cosmos, like the hearth, as a principle of equilibration,⁷⁶ and said it is called *Monas* because it is stable and remains (*menein*).⁷⁷ Psychologically, the number one signifies wholeness and integrity.⁷⁸

Von Franz explains that the Monad has two aspects, one as the all-inclusive *hen-to-pan*, the other as a unit in a multiplicity.⁷⁹ The Monad is unitary, but it becomes many through a kind of *kenôsis* (emptying), by which it becomes the units constituting a multitude.⁸⁰ From an all-comprehending wholeness emerge empty units; it gives up its individual quality to become quantity. The *Theologumena* says that there is one Monad with respect to form (*eidei*), but it produces many monads with respect to quantity (*megethei*).⁸¹ Similarly, the Gnostic Marcus refers to the All-one as *Monotês* and the numerical unit has *Henotês*,⁸² and Theon of Smyrna said the difference between the Monad and One was similar to that between number (*arithmos*) as intelligible form and numerable thing (*arithmêton*) as sensible unit.⁸³ Martianus Capella and Favonius Eulogius likewise distinguished the Monad as either *numerus* or as *numerabile*.⁸⁴ (Nevertheless, *monās* and *hen* were often used variably and inconsistently in the Platonic tradition.⁸⁵)

Psychologically, this double aspect of the Monad implies that it is the Dyad *in potentia*,⁸⁶ but that the Indefinite Dyad is required to split or double the Monad and to create the units.⁸⁷ This is why the Triad is the first odd number, for the Monad is, properly speaking, prior to the numbers.

⁷⁴ Jacobi (1967) 31.

⁷⁵ *TA* 3, 13–17.

⁷⁶ *TA* 6, 16–18.

⁷⁷ TA 1, 4–5.

⁷⁸ Lakoff and Núñez (2000) 75.

⁷⁹ von Franz (1974) 62.

⁸⁰ von Franz (1974) 62.

⁸¹ TA 2, 15–17.

⁸² von Franz (1974) 62–63n7, 90n18.

⁸³ Theon Smyrnaeus, *Exp. rerum math.* (Hiller) 19, 14. See also Kalvesmaki (2006) 285.

⁸⁴ Robbins (1921) 120n2.

⁸⁵ Kalvesmaki (2006) Excursus B.1.

⁸⁶ von Franz (1974) 90n18.

⁸⁷ von Franz (1974) 97.

III. Dyad

According to von Franz, the effect of the Dyad is to divide, to repeat, and to engender symmetries;⁸⁸ we will consider each.

To begin, the *Theologumena* calls the Dyad "division" and says it is the source of difference and inequality.⁸⁹ Similarly, according to von Franz, as a preconscious dynamism, the archetypal Dyad underlies division.⁹⁰ Indeed she cites Plato and the Pythagoreans in describing the Indefinite Dyad as the active principle that divides or doubles the Monad and creates a definite Dyad.⁹¹ As the *Theologumena*⁹² says, "when the Monad manifests unification, the Dyad steals in and manifests separation."⁹³ Karl Menninger likewise observes that polarities, such as male/female, right/left, and day/night, retain a strong element of underlying unity; therefore, separateness and togetherness are bound together in the Dyad.⁹⁴

The *Theologumena* compares the Dyad to daring (*tolman*) and impulse (*hormên*) because it has advanced into action;⁹⁵ it is the first to separate from the Monad⁹⁶ and is called movement and change.⁹⁷ According to von Franz, as a "dynamic pattern of human thought," the Dyad creates and symbolizes the many polarities of life.⁹⁸ Similarly, the *Theologumena* calls the Dyad "opinion" (*doxa*) because it contains both truth and falsity.⁹⁹ Psychologically the Dyad represents "the process of facing and enduring conflict."¹⁰⁰ This is a fundamental human experience, and the basal ganglia deep in the brain seem to be where alternative actions are subconsciously weighed and ultimately chosen. When on the horns of a dilemma, that uncomfortable feeling of suspense is part of the psychological experience of the Dyad.

By means of the Dyad, unconscious conflict, which is devoid of definite quality, becomes manifest in consciousness, which gives the conflict reality;¹⁰¹ therefore the Dyad also represents the transition to consciousness.¹⁰² According to Jung, "Conscious perception means

⁹³ *TA* 9, 6–7.

⁸⁸ von Franz (1974) 74.

⁸⁹ *TA* 8, 3; 11, 19–20.

⁹⁰ von Franz (1974) 88.

⁹¹ von Franz (1974) 63.

⁹² *TA* 11, 19–20.

⁹⁴ Menninger (1992) 13; see also von Franz (1974) 93–94n33.

⁹⁵ Waterfield (1988) 42; *TA* 9.6–7.

 $^{^{96}}$ TA 7, 20 – 8, 1.

⁹⁷ TA 8, 2–3.

⁹⁸ von Franz (1974) 89.

⁹⁹ TA 8, 1–2.

¹⁰⁰ Sparks (2010) 57.

¹⁰¹ Sparks (2010) 67, 87.

¹⁰² Sparks (2010) 63.

discrimination,"¹⁰³ and von Franz says, "Whenever a latent unconscious content pushes up into consciousness, it appears first as a twofold oneness."¹⁰⁴

Moreover, conscious manifestation and material manifestation are closely related, for they both involve the appearance of relatively discrete, discontinuous, stable objects from a background of undifferentiated wholeness.¹⁰⁵ Both literally and metaphorically, figures emerge from the background by means of the Dyad.

The Theologumena calls the Indefinite Dyad "matter," because it is "the source and foundation of the diversity of numbers."¹⁰⁶ As an active principle it is devoid of qualities, but it obtains definition from the Monad, and so the Dyad is called "Erato" because she attracts the Monad as form to her matter, thus generating all the results (apotelesmata) beginning with the Triad and Tetrad.¹⁰⁷ The *Theologoumena* also calls the Dyad "the fount of flowing and liquidity,"¹⁰⁸ common metaphors for the flux of material reality, and it says the Pythagoreans name it "Nature" because it has advanced into being from a seed principle (logou spermatikou).¹⁰⁹ Thus the Dyad is named "Rhea," for she is the mother of the gods and nature, and also because the Dyad is the origin of the flux — $rho\hat{e}$ — of material reality, and of *rhythmic* extension,¹¹⁰ for *rhuthmos* derives from *rheô* (to flow).¹¹¹

The *Theologumena* also says the Dyad is the first to separate from the Monad, and therefore it is the principle of linear extension and progression.¹¹² As the principle of difference, the Dyad is also called "infinity" (apeiron) since it begins the unending series that begins with its separation from the Monad.¹¹³ According to von Franz, the Dyad *repeats*, that is, it generates a translational or repetitive symmetry;¹¹⁴ it is the principle of duplication and reflection, leading to mirror symmetry, which is how the Dyad often manifests in visions, dreams, and art.¹¹⁵ Likewise, Pythagoreans call it "Equal" (isa).¹¹⁶

Von Franz says, "Considered as a rhythm of movement, the number two represents a repetition, in the form of an oscillation."¹¹⁷ Because, however, this is a simple oscillation

¹⁰³ Translated at von Franz (1974) 92.

¹⁰⁴ von Franz (1974) 93; see also 91–93.

¹⁰⁵ von Franz (1974) 94.

¹⁰⁶ Waterfield (1988) 41, 45. ¹⁰⁷ *TA* 13, 6–17.

¹⁰⁸ εὐπορίας καὶ χύσεως οὕσῃ πηγῃ (*TA* 8, 21 – 9, 1); Waterfield (1988) 42.

¹⁰⁹ *TA* 13, 14–17.

¹¹⁰ TA 14, 7–9.

¹¹¹ LSJ s.v. ῥυθμός.

¹¹² TA 12, 17 – 13, 1; 13, 6–11.

¹¹³ TA 12, 17–23.

¹¹⁴ von Franz (1974) 74. ¹¹⁵ von Franz (1974) 88.

¹¹⁶ TA 11, 1.

¹¹⁷ von Franz (1974) 94.

between two states, a back and forth, there is no inherent direction to it; it looks the same forward or backward.¹¹⁸ There is no answer to the Orphic enigma, "Which came first, the chicken or the egg?" Although this progression is extended in both time and space,¹¹⁹ actual forward direction comes with the Triad, which is nevertheless implicit in the extension of the Dyad.¹²⁰

Von Franz tells us that the Dyad creates a tension between the opposites that demands release;¹²¹ "duality seeks resolution in the third,"¹²² for the oscillation between the opposites is not pointless, but is seeking a synthesis.¹²³ It is the confrontation of the conscious Dyad with the originally unconscious Monad that gives birth to the Triad, which brings the needed resolution.¹²⁴ Thus psychologically the Triad emerges through consciousness of the Dyad, that is, the Triad arises from consciousness of duality as a whole.¹²⁵

IV. Triad

According to von Franz, the Triad is a manifestation of the Ineffable One as a knowable unit, which would have been impossible without the polarity of the Dyad,¹²⁶ for the tension of the opposites restores the unity in a knowable form.¹²⁷ Enduring the conflict inherent in the Dyad precipitates a synthesis of the opposed elements, which is a step toward a coherent and non-split personality. When the ego takes responsibility for the tension of the opposites, a resolution arises from the unconscious; it is given to us, not consciously crafted.¹²⁸ Therefore the Triad restores the lost unity,¹²⁹ and the *Theologumena* calls it *Harmonia* because it unifies the opposites.¹³⁰

The *Theologumena* also says that the Triad is the first number to actualize the potential of the Monad¹³¹ and the first to signify totality, because it comprises beginning, middle, and end;¹³² therefore it is the form of completion (*telesiourgias*).¹³³

¹¹⁸ von Franz (1974) 96.

¹¹⁹ Sparks (2010) 70; von Franz (1974) 97.

¹²⁰ von Franz (1974) 96–97.

¹²¹ von Franz (1974) 98.

¹²² Sparks (2010) 70.

¹²³ Sparks (2010) 56–57, 70–71, 88.

¹²⁴ von Franz (1974) 96.

¹²⁵ von Franz (1974) 64, 96.

¹²⁶ von Franz (1974) 98.

¹²⁷ Sparks (2010) 87.

¹²⁸ Sparks (2010) 55, 57, 73, 87–88.

¹²⁹ von Franz (1974) 98.

¹³⁰ *TA* 19, 18.

¹³¹ *TA* 14, 15–16; 16, 8–9.

¹³² *TA* 17, 5.

Ibn Gabirol said that the Triad is the root of the whole, for the Monad is form and the Dyad is matter, but the whole comprises both form and matter.¹³⁴ Therefore the *Theologumena* says the Triad is unique in being equal to the numbers that precede it, that is, 3 = 2 + 1, and so it synthesizes its predecessors.¹³⁵

The *Theologumena* also calls the Triad the manifestation of plurality and the first *actual* number because it is the first to be a system of units, the Monad and Dyad being roots or principles, not numbers.¹³⁶ Von Franz agrees, saying "The number series thus begins with three."¹³⁷ Both von Franz and the *Theologumena* observe that the existence of singular, dual, and plural grammatical numbers in Greek and some other languages indicates that the Triad is the first actual plurality.¹³⁸

Von Franz agrees with the Pythagoreans in seeing two series of numbers emanating from the Monad.¹³⁹ The odd numbers exhibit a centered structure, which focuses on a middle, and the even numbers have bilateral symmetry, which emphasizes polarity and opposition. Further, the Triad centers bilateral symmetries and initiates linear succession.¹⁴⁰ She says that when the Dyad is recognized on the background of the primordial Monad, the axis of symmetry becomes salient and becomes the third that reveals the Triad.¹⁴¹ Likewise, the *Theologumena* calls the Triad a *mean* because of its symmetrical relationship to opposed extremes of the same species.¹⁴² So also virtues are considered means between opposed extremes, which are excesses and vices. The vices are assigned to the Dyad, for they are indefinite and unknowable, but the golden mean is the actualized Monad, for it is definite and knowable.¹⁴³ Psychologically, the Triad centers the symmetric oppositions in the psyche and initiates a progression to a re-centered and stable personality.¹⁴⁴

Von Franz observes that the Triad is often associated with time.¹⁴⁵ Similarly, the *Theologumena* mentions the three parts of time: past, present, and future,¹⁴⁶ and says that every process partakes of the Triad in that it has three boundaries: beginning, peak, and end (*arkhên akmên teleuten*).¹⁴⁷ These three boundaries divide the process into two phases: increase and

¹³³ *TA* 17, 19.
¹³⁴ Quoted at von Franz (1974) 104n9.
¹³⁵ *TA* 14, 20; 15, 2–5, 15–16; 17, 17; 18, 4–5.
¹³⁶ *TA* 17, 15–17.
¹³⁷ von Franz (1974) 103.
¹³⁸ von Franz (1974) 106–107; *TA* 18, 11–13.
¹³⁹ von Franz (1974) 78–79.
¹⁴⁰ von Franz (1974) 74.
¹⁴¹ von Franz (1974) 64.
¹⁴² *TA* 15, 5–10.
¹⁴³ *TA* 19, 11–17.
¹⁴⁴ Sparks (2010) 56.
¹⁴⁵ von Franz (1974) 125.
¹⁴⁶ *TA* 17, 10.
¹⁴⁷ *TA* 16, 15.

decrease (*auxêsin kai phthisin*), and therefore implicate the Dyad.¹⁴⁸ In contrast to the oscillation of the Dyad, which is reversible and has no inherent direction, the triadic process has a direction; increase followed by decrease is different from decrease followed by increase. Therefore the Triad imparts direction to the process initiated by the Dyad, for it is oriented in the past, present, and future.¹⁴⁹ Since the Triad has a direction, it progresses psychologically in space as well as time.¹⁵⁰ Thus in the *Theologumena* the Triad is also associated with the three dimensions of space.¹⁵¹

Von Franz says the Triad symbolizes the psychodynamical process by which a "totality symbol" emerges "in a temporal succession so that it does not congeal into a static symmetry or harmony."¹⁵² She adds that "three signifies a unity which dynamically engenders self-expanding linear irreversible processes in matter and in our consciousness."¹⁵³ Likewise Sparks says, "The three is the dynamism of development inherent in our conflicting pieces, but it is not activated (i.e., is latent)... until we begin investigating what and who those pieces in us are."¹⁵⁴

The *Theologumena* says the Triad is called Prudence and Wisdom because it looks to the past, present, and future.¹⁵⁵ Psychologically, as a resolution of the dualistic conflicts, the real Self can emerge, "the birth of true coherence, substance, and individuality,"¹⁵⁶ according to Sparks. However this realization, which arose from the unconscious, was in von Franz' words, "reconstructed through discursive thought processes, and, in this process, became temporally conditioned."¹⁵⁷ Thus, timeless truths are understood discursively.¹⁵⁸

According to Jung, this flow of psychic energy is often associated with threefold underworld beings, indicating a connection to time and fate,¹⁵⁹ and the *Theologumena* associates the Triad with the Three Fates, who govern emission, receiving, and requital (*proeseôs*, *hupodokhês*, *antapodoseôs*).¹⁶⁰ Psychologically, fate manifests in a correspondence of inner and outer events, dual realities coordinated by a third that bridges them,¹⁶¹ and the *Theologumena* similarly tells us that the Triad unifies and harmonizes opposites, in particular the heavenly and earthly worlds.¹⁶² Therefore the triadic stage of the psychological process is often characterized

¹⁴⁸ *TA* 16, 16.

¹⁴⁹ TA 16, 18–22.

¹⁵⁰ von Franz (1974) 104.

¹⁵¹ TA 16, 15; 17, 10.

¹⁵² von Franz (1974) 109. See also Sparks (2010) 56, 88, 95; von Franz (1974) 104.

¹⁵³ von Franz (1974) 106.

¹⁵⁴ Sparks (2010) 95.

¹⁵⁵ TA 16, 18–22.

¹⁵⁶ Sparks (2010) 88.

¹⁵⁷ von Franz (1974) 125.

¹⁵⁸ von Franz (1974) 125–126.

¹⁵⁹ von Franz (1974) 104.

¹⁶⁰ *TA* 19, 7–8; see also von Franz (1974) 104n12.

¹⁶¹ Sparks (2010) 89–90.

¹⁶² *TA* 19, 5–11.

by synchronicities, that is, by meaningful coincidences between the inner and outer worlds.¹⁶³ We must be sensitive to *kairos*, for fate presents opportunities but we must respond at the right time.¹⁶⁴ Von Franz says that in myths, the hero may encounter a trio of challenges before arriving at a fourth, which signals the climax of development.¹⁶⁵

V. Tetrad

We come now to the Tetrad, which Von Franz says "acts as a stabilizer by turning back to the one as well as by bringing forth observables by creating boundaries."¹⁶⁶

In Jungian psychology as in the *Theologumena* the Tetrad represents wholeness and full completion (*apotelesma*).¹⁶⁷ Pythagoreans call the Tetrad the begetter of the Decad (*decados gennêtikên*), for the Decad is consummated by the Tetrad and its predecessors, that is: 10 = 4+3+2+1.¹⁶⁸ This is of course the Tetractys, which contains, according to the Pythagorean oath, "the fount that holds the roots of ever-flowing Nature."¹⁶⁹ The *Theologumena* tells us that these roots are the first four numbers: "the Monad of sameness which is regarded as absolute, the Dyad of difference and what is already relative, the Triad of particularity and of actual oddness, the Tetrad of actual evenness."¹⁷⁰

According to von Franz, "Jung devoted practically the whole of his life's work to demonstrating the vast psychological significance of the number four."¹⁷¹ The significance is that the Tetrad is a symbol of psychological wholeness and integration.¹⁷² One reason, as Jung explains, is that once unconscious content emerges into consciousness, it becomes subject to the four functions of consciousness: perception, thought, feeling, and intuition.¹⁷³ In Jungian psychology as in the *Theologumena* the Tetrad represents wholeness and embodiment.

Therefore the Tetrad leads back to the Monad, not as an unconscious undifferentiated plenum, but as a consciously articulated whole.¹⁷⁴ In this connection Jungians quote the

- ¹⁶⁹ *TA* 22, 21–22.
- ¹⁷⁰ *TA* 23, 4–9.

¹⁷² Sparks (2010) 57.

¹⁶³ Sparks (2010) 88–89.

¹⁶⁴ Sparks (2010) 90.

¹⁶⁵ von Franz (1974) 104.

¹⁶⁶ von Franz (1974) 74.

¹⁶⁷ Sparks (2010) 96; *TA* 20, 5, 11.

¹⁶⁸ *TA* 20, 5–7; 29, 5–6.

¹⁷¹ von Franz (1974) 115.

¹⁷³ von Franz (1974) 121; Jung (*CW* 10) ¶774.

¹⁷⁴ Sparks (2010) 60.

alchemical Axiom of Maria Prophetissa: "Out of the One comes Two, out of Two comes Three, and from the Third comes the One as the Fourth."¹⁷⁵

The *Theologumena* says that there are four basic numerical qualities: sameness in the monad, difference in the dyad, surface in the triad, and solidity in the tetrad.¹⁷⁶ Likewise, at the fourth psychological stage the Triad enters material reality and interacts with it. The previously personal revelation "is accordingly reshaped, modified, humanized, relativized."¹⁷⁷ The *Theologumena* says, "The tetrad is the first to encompass minimal and most seminal embodiment" (*sômatôsis elakistê kai spermatikôtatê*),¹⁷⁸ because the most elementary body is fire, which is shaped like a tetrahedron, which has four sides and four corners.¹⁷⁹ The fourth unit, which created the tetrahedron from the triangle and stands above it, can symbolize the more objective perspective that the Tetrad will bring to the triadic resolution.¹⁸⁰

The Tetrad represents a new level of psychological integrity for, as the *Theologumena* tells us, the tetrahedron is "hard to dissolve" (*dusdialuton*);¹⁸¹ it "binds everything in a pyramidal manner."¹⁸² The psychological goal is to integrate the multiplicity of the psyche into a new unity, in Spark's words, "a re-centered and stable personality."¹⁸³ As von Franz says, "the four acts as a stabilizer by turning back to the one."¹⁸⁴ Likewise, the *Theologumena* associates the Tetrad with Heracles because he is steadfast and often viewed as a moral hero.¹⁸⁵ Moreover, it calls the Tetrad Justice because four is a square and the only square whose area and perimeter are equal.¹⁸⁶

The Tetrad is the first evenly even number and the first properly square number, which also symbolizes its stability and security.¹⁸⁷ The Pythagoreans say the Tetrad is called "the enduring one" (*tetlada*),¹⁸⁸ because its square root, the Dyad, was the first to endure separation from the Monad, and because the Tetrad is the cause of the three spatial dimensions.¹⁸⁹ The *Theologumena* also associates the Tetrad with four measures of change: eternity, time, critical

¹⁷⁶ *TA* 25, 14–16.

¹⁷⁷ Sparks (2010) 96.

¹⁷⁹ *TA* 23, 12–16.

- ¹⁸⁰ Cf. Sparks (2010) 98.
- ¹⁸¹ *TA* 26, 6.
- ¹⁸² Waterfield (1988) 61.
- ¹⁸³ Sparks (2010) 56.
 ¹⁸⁴ von Franz (1974) 74.
- 185 TA 28, 1–2, 18–20.
- ¹⁸⁶ TA 29, 6–8.
- ¹⁸⁷ *TA* 28, 2–4.

 $^{^{175}}$ von Franz (1974)
65; Jung (CW12) ¶26.

¹⁷⁸ Waterfield (1988) 57; *TA* 23.11–12.

¹⁸⁸ *TA* 29, 2.

¹⁸⁹ TA 29, 4–5.

time, and passing time (*aiôn*, *khronos*, *kairos*, *hôra*),¹⁹⁰ therefore it brings the transcendent Triad into time and space.

Von Franz says that the "difficult step" from the Triad to the Tetrad is "the progression from the infinitely conceivable to finite reality,"¹⁹¹ because the Triad represents the fated pattern of one's life, "genuine possibility," the "transcendental continuum," which has been made conscious, but the Tetrad seeks to actualize it in material reality, in one's actual embodied life.¹⁹² According to Sparks, the Tetrad represents a genuine selfhood that is accessible and open to outside influences, emotionally available, and humane.¹⁹³

To some extent, the psychological difference between the Triad and the Tetrad is the difference between the generic and the individual. The triadic stage involved the conscious recognition of universal truths from the collective unconscious, but at the tetradic stage we realize that they are necessarily grasped by finite, embodied, historical human beings. Therefore the sequence of the first four archetypal numbers define four levels of consciousness:¹⁹⁴

- I. Uncritical unconsciousness
- II. Dualistic: tension, doubt, criticism
- III. Gnosis: unity in higher world
- IV. Ego as herald of universal truths

The insights and inspirations arising from the unconscious must be interpreted and explained using the time-bound discursive process of conscious thought; this is the fourth level.¹⁹⁵ In this way, according to Sparks, "we can both have and detach from our own 'divine truth', *and* enter into open conversation and productive self-reflection."¹⁹⁶

The tetradic level of consciousness recognizes that although the triadic insights come as divine revelations, they are modified by individual conscious interpretation. The ego therefore becomes the interpreter¹⁹⁷ and assumes the role of Hermes, messenger of the gods, whom the *Theologumena* identifies with the Tetrad, for he was commonly represented on four-sided herms¹⁹⁸ and was born on the fourth day of the fourth month, according to tradition.¹⁹⁹

¹⁹⁰ TA 24, 12–18.

¹⁹¹ von Franz (1974) 122.

¹⁹² Sparks (2010) 101–102.

¹⁹³ Sparks (2010) 98–99.

¹⁹⁴ Sparks (2010) 105–106; von Franz (1974) 124–125.

¹⁹⁵ von Franz (1974) 125–126.

¹⁹⁶ Sparks (2010) 108, emphasis in original.

¹⁹⁷ Sparks (2010) 106.

¹⁹⁸ *TA* 28, 2–4.

¹⁹⁹ That is, the month Hermaios in Argos; see Kerényi (1995) 56–57.

Although the triadic truths are timeless, they are reconstructed in a temporal context by time-bound discursive thought.²⁰⁰ At the triadic level we were certain of the universality of the triadic synthesis, but at the tetradic level we understand that these truths have to be relativized to *our* time and place.²⁰¹ We learn to separate their apparent absolute validity from our unavoidably subjective interpretation; Truth with a capital "T" is replaced by "my truth."²⁰²

Nevertheless, since our subjective conscious interpretation is faced with an objective unconscious, we must engage it through means such as dream interpretation and active imagination;²⁰³ that is, theurgy.

Advancing from the Triad to the Tetrad requires acknowledging and integrating all four functions of consciousness. At the triadic level, we are comfortable relying on our dominant function and its two auxiliary functions, but at the tetradic level we must recognize and integrate our inferior function, and accept the gifts it has to offer.²⁰⁴ As Alain Negre says, "Its integration into consciousness corresponds to the passage from Three to Four, allowing a return to the One, not through a regression to the fusional state but in a fully differentiated plane of consciousness."²⁰⁵ In particular, the fourth level requires that we embrace our Shadows, personal and collective, so we may recognize their intervention in our lives and recruit them to fulfill our destinies.²⁰⁶ All of these reflections of the 1+3 structure of the Tetrad are symbols by the fact that the Tetrad is the sum of the first two triangular numbers, the Monad and the Triad.²⁰⁷

The double opposition of the square immediately brings its center to our attention, and the centered square or quincunx has a special meaning, for it shows the Tetrad both as emergent from the Monad and as an articulated Monad.²⁰⁸ Jung points to the Western alchemical tradition in which the centered square represents the quintessence in the center of the elemental square.²⁰⁹ Von Franz says the *quinta essentia* "represents the most refined, spiritually imaginable unity of the four elements."²¹⁰ Recalling the Axiom of Maria, Sparks says, "The four is the same as the one—a singular, solid, and unique individual who is open."²¹¹ The quincunx and the quintessence both point us toward the Pentad, but the rest of the Decad lies outside the scope of this paper.

²⁰⁰ von Franz (1974) 125.

²⁰¹ Sparks (2010) 104, 106.

²⁰² Sparks (2010) 102–103; von Franz (1974) 126.

²⁰³ Sparks (2010) 103–104.

²⁰⁴ von Franz (1974) 127–129.

²⁰⁵ Negre (2018) 67.

²⁰⁶ von Franz (1974) 128, 133–134; Sparks (2010) 107–108.

²⁰⁷ von Franz (1974) 113–114.

²⁰⁸ von Franz (1974) 120–121, 124.

²⁰⁹ von Franz (1974) 120–121.

²¹⁰ von Franz (1974) 120–121.

²¹¹ Sparks (2010) 108n93.

VI. Conclusions

In conclusion, I've argued that numbers are not mere quantities and that the first four numbers, in particular, have rich qualitative structures; they are psychologically potent and numinous. Jung and the Jungians have explored these archetypal numbers, which have psychodynamical properties common to all people, for they are rooted in the human brain. These properties agree with many of the qualities of the archetypal numbers described in the *Theologumena Arithmeticae* and other traditional arithmological sources. Therefore a complete Platonic philosophy of mathematics should include the interior, psychodynamical qualitive structure of the numbers as well as their external, formal quantitative structure. The numbers objectively exist, independent of our conscious construction, as potent forces in our psyches and as governors of our lives.

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